

Turning points of massive particles in Schwarzschild geometry

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The stable geodesics in Schwarzschild geometry can not approach the center closer than the radius of the photon sphere, $3/2$ times the Schwarzschild radius. In other words, massive particles moving along geodesics that cross the photon sphere do not escape, they fall into the black hole.

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It is well known that there is no return for objects once they cross the horizon of the Schwarzschild geometry, namely when $r < r_s = 2GM/c^2$. But it is less known that there is a 50% larger critical distance, the radius of the photon sphere, [1] (see also [2]), corresponding to the circular orbits of massless particles. It is shown below that already the photon sphere radius is a point of no return for massive particles' geodesics.

The radial component of the velocity of a geodesics in Schwarzschild geometry takes the following form

$$(u^r)^2 = \epsilon - 2V_{\text{eff}}(r) \quad (1)$$

where we introduced an effective potential

$$V_{\text{eff}}(r) = \frac{g_{tt}}{2} \left(\frac{J^2}{r^2} + 1 \right) \quad (2)$$

per unit mass, $g_{tt} = 1 - r_s/r$ and the integration constants ϵ and J arise due to the conservation of energy and angular momentum, respectively. This is a monotonically increasing function of r for small angular momentum, $J^2 < 3r_s^2$, but it develops local minimum $V(r_+)$ and local maximum $V(r_-)$ at

$$r_{\pm} = \frac{J^2}{r_s} \left(1 \pm \sqrt{1 - \frac{3r_s^2}{J^2}} \right) \quad (3)$$

for larger values of angular momentum, $J^2 > 3r_s^2$, cf. Fig 1. The positions of local minimum, r_+ , and local maximum, r_- , are found to be monotonic functions of J^2 , increasing and decreasing, respectively and we have $r_+ \rightarrow \infty$ and $r_- \rightarrow r_-^\infty = 3r_s/2$ as $J \rightarrow \infty$.

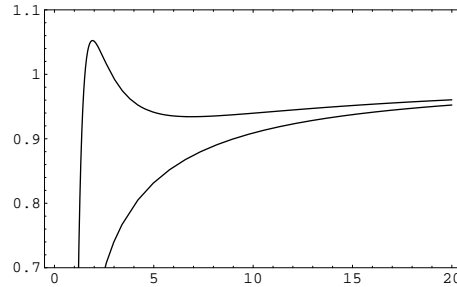


FIG. 1: The effective potential of Eq. (2) as the function of the dimensionless ratio r/r_s for $J/r_s = 2.1$ (upper curve) and $J/r_s = 1$ (lower curve).

The inner turning point on the geodesics, r_{\min} , satisfying the equation

$$\epsilon - 2V_{\text{eff}}(r_{\min}) = 0 \quad (4)$$

must be on the decreasing part of the effective potential, considered as the function of the radius. Therefore, r_{\min} can not be arranged closer to the center than $r_{\min} = r_-^\infty$ i.e. within the photon sphere. One can say that the photon

sphere is impenetrable for massive geodesics: once the freely falling massive particle crosses the photon sphere, it will not escape that region anymore (in the case of a black hole, it will inevitably reach the Schwarzschild radius).

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- [1] M. A. Abramowicz and J. P. Lasota, Acta Phys. Polon. **B5**, 327 (1974).
 - [2] M. A. Abramowicz, S. Bajtlik and W. Kluzniak, Phys. Rev. A **75**, 044101 (2007)